Towards reasoning over knowledge graphs under aleatoric and epistemic uncertainty

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***Abstract*—The objective of this short article is to propose a research orientation to address the international Knowledge Graph Reasoning Challenge (IKGRC). The IKGRC consists in solving mysterious crimes and providing a reasonable explanation based on open knowledge graphs, which represent the case, background, and characters of some Sherlock Holmes novels.**

The suggestion of this paper is to use the framework of the belief functions, well adapted to the consideration of different uncertainties and the management of conflicting information.

***Index Terms*—Knowledge Graph, Belief Functions, Treatment of Uncertainties**

I. Introduction

Different types of uncertainty can disturb reasoning over a knowledge graph: a) aleatory uncertainty, i.e., intrinsic to the hazard of a phenomenon, or b) epistemic uncertainty, i.e., resulting from a lack of knowledge or data. Let us note that conflict between several sources is one possible cause of uncertainty. In the IKGRC[1], epistemic uncertainty can arise from the incompleteness of the graph, low granularity or imprecise statements, like a partial description of the culprit in witness testimony. On the other hand, aleatoric uncertainty arises when some statements can be untrue. An adapted framework could allow us to take greater advantage of information affected by uncertainty.

II. Reasoning under uncertainty

In the past, several frameworks have been proposed to represent and deal with uncertainty such as the probability theory, the imprecise probabilities, the belief function theory, the possibilities, or the fuzzy logic. Some of these frameworks have been used for reasoning over a knowledge graph. For example, several works focus on reasoning over Knowledge graphs in the presence of fuzziness, i.e., variables may be true to a certain degree comprised in [0,1]. In [1] an approach is proposed to achieve reasoning with general terminological axioms in fuzzy description logic. On another note, [2] provides a logical query embedding framework for answering complex logical queries on knowledge graphs. In the framework of classical probabilities, [3] presented a Markov logic network-based approach for reasoning over uncertain temporal knowledge graphs.

[1]https://ikgrc.org/2023/index.html

To our knowledge, there is not yet a knowledge graph reasoning approach proposed in the framework of the belief functions. However, such an approach could allow for jointly managing both types of uncertainty. Moreover, this framework has rich literature on the subject of information fusion, and thus allows for possible ways of reasoning in the presence of conflicting information.

Let us briefly introduce the belief functions theory, also called Dempster Shafer theory [4], [5]. This theory has been used in various domains including the semantic web [6], and ontology [7]. Due to the additivity constraint inherent to the definition of a probability distribution, one cannot build a unique probability distribution when measures, observations, etc. imprecise, i.e., are affected by epistemic uncertainty. Belief functions theory, as an extension of probability theory, allows masses to be assigned to imprecise data or sets. Denoting the universe by Θ, a *mass function*, also called *basic belief assignment* (bba), is a set function $m : 2^\Theta \to [0, 1]$ satisfying

$$\sum_{A \subseteq \Theta} m(A) = 1. \tag{1}$$

For a set $A \subseteq \Theta$, the quantity $m(A)$ is interpreted as the mass of belief allocated exactly to the set A and not to more specific subsets of A. Let us illustrates the meaning of a mass function through an example borrowed from [8] originally inspired by [9]. We suppose that a murder has been committed and that there are three suspects: *Peter, John,* and *Mary*. We set then $\Theta = \{Peter,\ John,\ Mary\}$. We further suppose that a witness has seen the culprit running away. Since the witness is short-sighted, he could only testify that the culprit is a man (epistemic uncertainty). Moreover, this witness was drunk at the time which makes this evidence true with reliability 0.8 (aleatoric uncertainty). The evidence provided by this witness is then represented with the mass function defined by $m_1(\{Peter,\ John\}) = 0.8$, $m_1(\Theta) = 0.2$. A mass function induces two other set functions that can be used to make inference. First, the belief function $Bel : 2^\Theta \to [0, 1]$, which quantifies the total belief in A as the sum of all masses of subsets of A: $Bel(A) = \sum_{B \subseteq \Theta, B \subseteq A} m(B)$. Second, the plausibility function of A, $Pl : 2^\Theta \to [0, 1]$, which quantifies the maximum mass that could be allocated to A: $Pl(A) = \sum_{B \subseteq \Theta, B \cap A \neq \emptyset} m(B)$.

One of the most important benefits of the framework of belief functions is the ability to combine different information from several sources. The most popular combination procedure used in the framework of belief functions is Dempster's combination (conjunctive combination) which assumes that all sources of information are independent and reliable and completely ignores the conflict between them. In order to combine two mass functions m_1, m_2, the Dempster combination, denoted $\oplus$ is defined as follows

$$m_1 \oplus m_2(A) = \begin{cases} 0 & \text{if } A = \emptyset, \\ \dfrac{1}{1-K} \displaystyle\sum_{B \cap C = A \neq \emptyset} m_1(B) m_2(C) & \text{elsewhere,} \end{cases} \tag{2}$$

where the $K = \sum_{B \cap C = \emptyset} m_1(B) m_2(C)$ measures of the amount of conflict between m_1 and m_2. Let us continue the example from [8]. Now we suppose that a blond hair was found on the crime scene and that the room was cleaned before the crime with a probability 0.6. This new piece of evidence is represented by the following mass function m_2: $m_2(\{John, Mary\}) = 0.6, m_2(\Theta) = 0.4$. When combining the two pieces of evidence with Dempster's rule, we obtain $m(\{John\}) = 0.48$, $m(\{Peter, John\}) = 0.32$, $m(\{John, Mary\}) = 0.12$, $m(\Theta) = 0.08$. Let us note that this combination tightens the mass on the intersections (loss of imprecise information). This combination can therefore be risky, especially when the degree of conflict is important. One other very used masses combination is the disjunctive combination, which also assumes that the sources of information are independent, but which is suitable for situations where at least one source of information is reliable. Other combinations have been proposed to meet particular situations of information sources. Let us cite the conjunctive combination of Smets [10], the conjunctive combination of Yager [11], the disjunctive combination of Dubois [12] etc.

To benefit from the many modeling and inference tools of the belief functions framework in the situation where sources of information are given by a knowledge graph, we suggest relying on the *basic theory of Uncertain Logic Processing* (ULP) presented in [13]. ULP permits the management of information given in the form of first-order logic formulas subject to uncertainty, where the uncertainty is expressed in the belief functions framework. In the same article, the authors suggest a way to extend the concept of the satisfiability (SAT) problem into ULP.

The idea for reasoning in the belief functions framework over knowledge graphs is the following. First, the relevant triplets in the graph are extracted and interpreted as uncertain first-order logic formulas. Then ULP is used to generate mass function assignments based on the uncertain first-order logic formulas. To do so, the uncertainty associated with each formula should be made explicit. To us, capturing this uncertainty directly from the knowledge graph is still an open question. However, one could use external knowledge such as "If the witness knows the person he/she saw, then he/she is right with a probability p" (aleatoric uncertainty). Concerning the consideration of epistemic uncertainty, we could take advantage of an external ontology to capture the level of precision of an object or a subject.

One more global idea is to label each triplet with a source tag like "testimony of Helen" or "observation at the scene of the crime". Then, using the belief function framework, identify which sources, e.g., characters, are the most in conflict with the others or inconsistent with the facts. The labeling of the sources could be achieved by NLP or machine learning on the original sentences in the novels.

III. Conclusion

The idea presented in this paper is to propose an approach in the framework of belief functions allowing to represent and manage both aleatoric and epistemic uncertainty when reasoning on a knowledge graph.

Being able to take into account these two types of uncertainty could be useful to improve the expressiveness of reasoning models on knowledge graphs. The IKGRC problem is all the more adapted to this research direction, as it implies many sources of information to be processed and confronted to deduce the solution.

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